May 2, 1861.

Major-General SABINE, R.A., Treasurer and Vice-President,
in the Chair.

In accordance with the Statutes, the names of the Candidates re-
commended by the Council for election into the Society, were read
from the Chair, viz.—

Charles Spence Bate, Esq.
Heinrich Debus, Esq.
Campbell De Morgan, Esq.
Thomas A. Hirst, Esq.
A. Matthiessen, Esq.
J. Clerk Maxwell, M.A.
Ferdinand Müller, M.D.
William Newmarch, Esq.

Edmund Alexander Parkes, M.D.
William Pole, Esq.
Philip Lutley Sclater, M.A.
Charles Frederick Alexander
  Shadwell, Capt. R.N.
Henry J. Stephen Smith, M.A.
William Stokes, M.D.
George Johnstone Stoney, M.A.

The following communications were read:—

I. "On Internal Radiation in Uniaxal Crystals." By BALFOUR
STEWART, Esq., A.M. Communicated by J. P. GASSIOT,
Esq. Received April 11, 1861.

(Abstract.)

The well-known theory of exchanges, which was proposed by the
late Prof. Prevost of Geneva, is built upon the fact that a substance
placed anywhere within an enclosure of a constant temperature will
ultimately attain the temperature of the enclosure.

In his theory M. Prevost supposes that a constant, mutual, and
equal interchange of radiant heat takes place between the body and
the enclosure which surrounds it, so that, receiving back precisely
that heat which it gives away, the former is enabled to remain at a
constant temperature.

With respect to this radiation, which is thus supposed to be con-
stantly taking place between substances at the same temperature, it
had until lately been conceived of as proceeding mainly, if not en-
tirely, from the surface of bodies—a very thin film or plate of any

substance being supposed to furnish the maximum amount of radiation which that substance was capable of affording.

It lately occurred to the author of this paper, reasoning from the theory of exchanges, that mere surface radiation is not sufficient to account for the equilibrium of temperature which exists between a body and the enclosure which surrounds it.

These theoretical conclusions have been amply verified by experiment, and the subject has been discussed in a paper published in the 'Transactions of the Royal Society of Edinburgh' for the year 1858. As the chain of reasoning by which this fact is deduced theoretically from the law of exchanges, and the experimental evidence upon which it rests, are both of a very simple nature, it has been thought well to restate them here before proceeding further in this investigation.

Let us imagine to ourselves an enclosure of lamp-black kept at a constant temperature, and containing two pieces of polished rock-salt similar to one another, except that the thickness of the one is greater than that of the other.

Now it is evident that since the thick piece absorbs more of the heat which falls upon it from the sides of the enclosure than the thin piece, it must likewise radiate more in order that it may always remain at the same temperature. Here then we have the fact of internal radiation in the case of rock-salt deduced as a theoretical consequence of the law of exchanges ; experimentally it is found that a thick piece of rock-salt radiates very considerably more than a thin piece.

The fact of internal radiation being conceded, it is easy to see that the amount of heat which a particle radiates must be independent of its distance from the surface. For besides that this is the simplest hypothesis, the absorption, and consequently the radiation of two similar plates of rock-salt placed with their surfaces together, ought to be the same as from a single plate of double the thickness ; and experiment shows that this is the case.

It being therefore supposed that the internal radiation of a particle is independent of its distance from the surface, let us imagine a row of particles A, B, C, D in the midst of a substance of constant temperature which extends indefinitely on all sides of them. There will be a certain stream of radiant heat constantly flowing past any such particle A to go in the direction AB.

Now, since the radiation is supposed to be the same for the different particles A, B, C, D, it follows that the absorption of the stream of heat by these particles must also be the same for each; and in order that this may be the case, it is necessary that the stream which impinges on one particle be the same in quantity and in quality as that which impinges upon another. This consideration leads us to a method of viewing internal radiation, which is wholly independent of the diathermanous or athermanous character of the body. For whatever be the absorption of a particle for any description of heat, its radiation must necessarily be precisely the same in order that the stream of heat in passing the particle may be just as much recruited by its radiation as it is reduced by its absorption; in other words, we may regard the substance through which the heat passes as perfectly diathermanous.

We gain another advantage by this method of viewing the subject: for, in the law which is expressed by saying that the absorption of a particle is equal to its radiation, and that for every description of heat, the word *description* is used to define and separate those rays of heat which are absorbed in different proportions by the same substance. Therefore in any problem connected with this subject we may suppose that a separate equilibrium holds for every such ray.

Now it is well known that rays of different wave-lengths are absorbed in different proportions by the same substance. We are therefore entitled to suppose that a separate equilibrium holds for each wave-length. The advantage of this is obvious in problems which admit of the application of optical principles. But we may go even further. For we know that in tourmaline, and in some other crystals cut parallel to the optic axis, the ordinary ray is more absorbed than the extraordinary; and the experiments of Prof. Kirchhoff and the author have shown that in tourmaline the ordinary ray is also radiated in excess. It thus appears that, in the case of crystals, we have not only a separate equilibrium for each wave-length, but for each of the two rays into which the incident ray is divided.

The following method of comparing together two streams of radiant heat has been adopted:—Consider a square unit of surface to be placed in the midst of a solid of indefinite thickness on all sides, and find the amount of radiant heat which passes across this square unit of surface in unit of time in directions very nearly perpendicular to

the surface, and comprehending an exceedingly small solid angle $\delta\phi$. Call this heat $\mathrm{R}d\phi$, then R may be viewed as the intensity of the radiation in this direction.

Let us now suppose that we have a uniaxal crystal of indefinite thickness bounded by a plane surface, and that parallel to this surface, and separated from it by a vacuum, we have a surface of lamp-black, the whole being kept at a constant temperature.

Let us take a square unit of this surface, and consider the heat from the lamp-black which falls upon it through an exceedingly small solid angle in a direction not necessarily perpendicular to the surface. Part of this heat will be refracted into the interior of the crystal in two rays, the ordinary and the extraordinary. There will be thus two separate bundles of refracted rays, the solid angle comprised by the individual rays of the one being different from that comprised by the rays of the other ; the inclination to the surface also being different for each bundle.

Now, on the principle of a separate equilibrium for each ray, these entering bundles of rays must respectively equal the rays of the same kind which emerge from the crystal in the same directions.

Hence if we know the radiation of lamp-black, and the direction in which the rays under consideration strike the surface of the crystal, as also the angle which the latter makes with the optic axis, it is conceivable that, by means of optical principles, joined to the fact of the equality between the entering and emerging bundles of rays, we may be enabled ultimately to ascertain the internal radiation through the crystal in different directions.

A little consideration, however, will show that this method of procedure presupposes a certain mutual adaptation to exist between the optical principles employed and the theory of exchanges. For it is evident that the expression for the internal radiation in any direction may be obtained by operating upon terminal surfaces bearing every possible inclination to the optic axis.

But the internal radiation, if the law of exchanges be true, is clearly independent of the position of this surface, which is indeed merely employed as an expedient. This is equivalent to saying that the constants which define the position of the bounding surface must ultimately disappear from the expression for the internal radiation.

The author then endeavours to show that such an adaptation does

really exist, and that the expression for the internal radiation is independent of the position of the surface.

For the extraordinary ray, the internal radiation is found to be

$$R_e = \frac{R r^4}{2 m^4 n^2},$$

where R is the radiation from lamp-black;

and for the ordinary, $\qquad R_0 = \frac{R}{2 n^2}$;

where n denotes the axial and m the equatorial radius of the ellipsoid into which the extraordinary ray will have spread in the crystal in the same time that *in vacuo* it would have spread into a sphere whose radius = unity; and lastly, r denotes the radius of this ellipsoid in the direction in which the internal radiation is measured.

The author concludes by remarking that the fundamental law, which is intimately connected with the theory of exchanges, and which renders an equilibrium of temperature possible in the case under consideration, seems to be the law of the equality between action and reaction in the impact of elastic bodies.

He also considers that the law which is expressed by saying " That the absorption of a particle is equal to its radiation, and that for every description of heat," expresses another law of action and reaction which holds when the motion which constitutes radiant heat is not conveyed from particle to particle without loss, or when the bodies under consideration are not perfectly elastic.

These two laws of action and reaction are viewed as supplementing each other, so as to render that equilibrium of temperature which is demanded by the theory of exchanges possible under all circumstances.

II. " On Fermat's Theorem of the Polygonal Numbers." By the Right Hon. Sir Frederick Pollock, Lord Chief Baron. Received July 11, 1860. Revised by the Author April 25, 1861.